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STRUCTURAL INSTABILITY OF INVISCID TRANSONIC CHANNEL FLOW

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Transonic flow past a step located on the lower wall of a channel and modeling the airfoil is considered. The stability of stationary flow to small changes in the Mach number at the outlet of the channel is investigated numerically. The existence of special regimes in which flow is unstable, i.e., insignificant perturbations of the boundary conditions cause a qualitative change in its structure, has been established.

Airfoils having a small curvature in the central part are characterized by a high sensitivity of the flow pattern to a change in the parameters of the incoming flow [1–4]. It has been shown in [5] that this phenomenon is largely determined by the structural instability occurring under certain boundary conditions. Consideration was given to the airfoil

$$y(x) = y_{max}(1 - |2x - 3|^3), \quad 1 \le x \le 2, \tag{1}$$

in the channel $0 < x < 3$ and $0 < y < 1$ with a prescribed Mach number M'_{out} in the outlet cross section. The existence of the singular value M_s of this parameter causing restructurization of the flow was established. Two supersonic zones were observed near the airfoil when $M'_{out} < M_s$; they varied continuously with increase in M'_{out}. If M'_{out} was insignificantly higher than M_s, these two zones abruptly coalesced into one zone. As M'_{out} increased further, the flow varied continuously again. If M'_{out}, being initially higher than M_s, decreased and became lower than the value M_s, the supersonic region, conversely, broke down abruptly into two zones.

Thus, it has been shown that beyond a small vicinity of the singular value M_s, transonic flow is stable to small changes in the boundary conditions. At the same time, as M'_{out} becomes lower than M_s, we have restructurization of the flow, which is caused by the impossibility of the intermediate stationary pattern of flow with two supersonic zones having one common point on the airfoil. The reason for the impossibility of this pattern is that *F*, i.e., the point of intersection of a shock wave closing the first supersonic zone and the airfoil, cannot coincide with the initial point A of the second supersonic zone (Fig. 1a) since the closing shock wave approaches the airfoil in the normal direction and the velocity of the flow behind it must necessarily be subsonic.

In this work, we study unstable regimes of transonic flow near an airfoil whose curvature has a minimum at the central point but does not vanish, unlike [3–5].

Formulation of the Problem. We consider an inviscid two-dimensional flow of a compressible gas in the channel $0 < x < 3$ and $0 < y < 1$ with parallel walls. On the lower wall, there is a small smooth step modeling an airfoil and determined by the polynomial

$$y(x) = y_{mid}[1 - p(2x - 3)^2 - q(2x - 3)^4], \quad 1 \le x \le 2, \quad p + q = 1. \tag{2}$$

It is obvious that $y(1) = y(2) = 0$ and the curvature of the central part of the airfoil increases with increase in the parameter p and decrease in $q = 1 - p$. When $p = 1$, the airfoil (2) coincides with a parabola passing through the points $x = 1$ and $x = 2$ of the x axis and having its peak at the point $x = 1.5$, $y = y_{mid}$.

Inviscid-gas flow is described by a system of Euler equations for the density $\rho(x, y, t)$, the velocity components $u(x, y, t)$ and $v(x, y, t)$, and the internal energy of a unit volume $e(x, y, t)$. The setting of the boundary condi-

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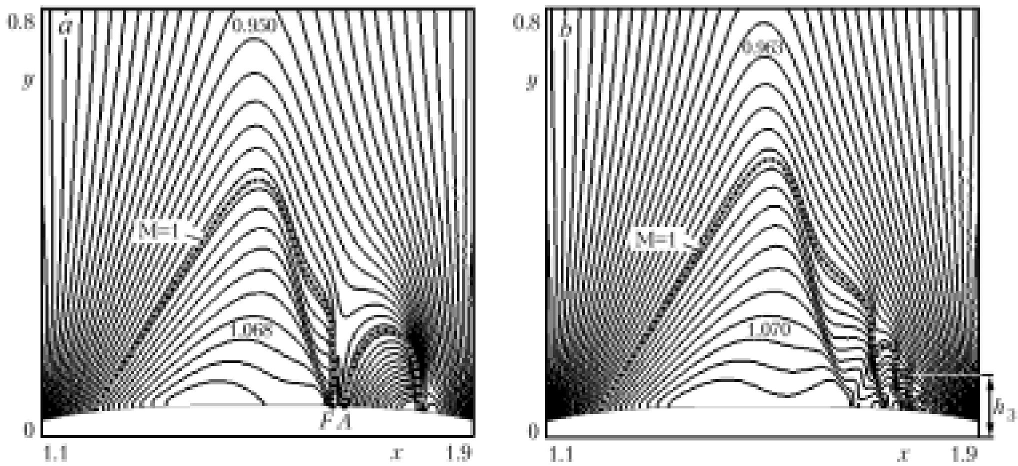

Fig. 1. Fragment of transonic channel flow. Lines of constant values of the Mach number near the airfoil (2) with p = 0.3 and y_{mid} = 0.06: a) M'_{out} = 0.7439 and b) 0.7441.

tions in the problem in question is the same as in [6, 7]: the nonflow condition is specified on the channel walls, the static pressure in the flow p_{out} is prescribed at the outlet of the channel, and the zero value of the vertical velocity component $v(0, y, t) = 0$ and the values of the entropy and enthalpy are prescribed at the inlet of the channel. The values of the parameters at the initial time $t = 0$ are additionally prescribed in calculating nonstationary flow.

It is convenient to characterize the static pressure p_{out} at the outlet of the channel by the arbitrary Mach number M'_{out} obtained from the isentropic relation $p_0/p_{\mathrm{out}} = [1 + (\gamma - 1)\, M'^{2}_{\mathrm{out}}/2]^{\gamma/(\gamma-1)}$, where the total pressure p_0 is determined by the boundary conditions at the inlet of the channel.

To numerically solve the problem formulated we employed the same method as in [4, 8]. The nonstationary solution was calculated using an ENO2 difference scheme of second order of accuracy [9]. The nonuniform computational grid was formed by vertical straight lines and lines produced by subdivision of the variable channel width into a fixed number of steps. Their size $\Delta x = \Delta y$ was constant in the central part of the channel and it increased in the direction of the upper wall and in the region of subsonic flow near the inlet and outlet cross sections of the channel.

In the series of calculations carried out, the relaxation of the flow and its reaching the stationary regime were observed after $4 \cdot 10^4$–$1.8 \cdot 10^4$ time steps depending on the initial conditions and the value of p in (2), i.e., on the airfoil selected. We employed a 401 × 171 grid with a step of $\Delta x = \Delta y = 0.004$ in the central part of the channel. The calculations on a finer (601 × 341) grid showed that it does not yield a substantial improvement in the accuracy of determination of the regimes of structural instability; however, the time it takes to compute the stationary solution increased by almost an order of magnitude. On the other hand, the employment of computational grids with larger steps led to an appreciable error in computation of the position of shock waves and singular Mach numbers, particularly in the variants where the size of supersonic regions was comparable to the size of several cells of the grid.

Results of Numerical Modeling. Figure 1a shows a fragment of the transonic flow obtained near the airfoil (2) with p = 0.3 for M'_{out} = 0.7439. As is seen, in this case there are two local supersonic zones that are $d \approx 0.06$ apart. The length of the segment *FA* indicated in Fig. 1a is taken as the distance d between the zones. The compression wave coming from the center of the airfoil due to the minimum of its curvature intersects the shock wave and causes a downstream departure of a sonic line from it. This line then bends in the direction of the airfoil; there forms a second shock wave arranged almost perpendicularly to the flow and closing the supersonic wave.

As M'_{out} decreases, the qualitative pattern of flow remains the same as in Fig. 1a; only the distance d somewhat increases. However, with an increase of 0.0001 in M'_{out} = 0.7439 the flow becomes unstable. The shock wave closing the first supersonic region shifts downstream and reaches the sonic line beginning at point A with the resulting coalescence of the supersonic zones. In the united region, we observe multiple reflection of the skew shock wave from the airfoil and from the line $M(x, y) = 1$ separating the supersonic region from the subsonic region (Fig. 1b). The flow field obtained is presented in Fig. 1b for M'_{out} = 0.7441. We observe the deflections of this line at reflection points, which corresponds to flow diagrams including the points of its return or inflection that have been considered in [3].

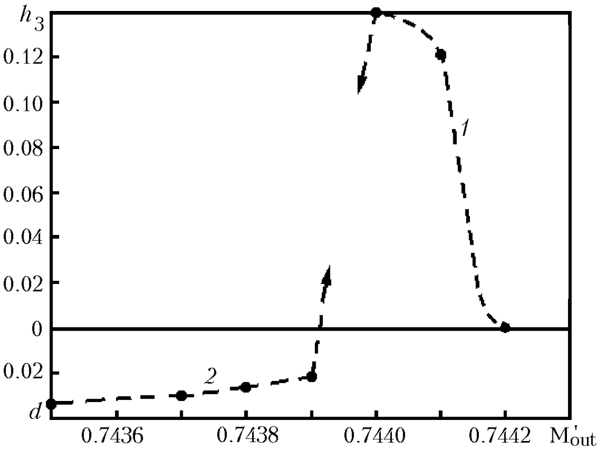

Fig. 2. Height h_3 (1) and distance d (2) vs. M'_{out} for the airfoil (2) with p = 0.3 and y_{mid} = 0.06.

As M'_{out} increases further, the qualitative pattern of flow changes insignificantly. We emphasize that both flow fields presented in Fig. 1 are stable to small perturbations of M'_{out} of the order of 10^{-5} and of the order of 10^{-4} if the value of the Mach number does not become lower than M_s = 0.7440. The sensitivity of the flow pattern to a change in M'_{out} can be characterized by a change in the height h_3 of the third deflection in the shape of the supersonic region when $M'_{out} > M_s$, and in the distance d between the supersonic zones when $M'_{out} < M_s$ (Fig. 2).

The analysis of unstable regimes of transonic flow for higher values of the parameter p in formula (2) has shown that the height of the supersonic subregions decreases and up to three (in stationary regimes) or even four (in nonstationary transitions) supersonic subregions can be realized instead of two local supersonic zones. Accordingly, several singular values of M'_{out} which lead to an unstable flow and an abrupt change in the flow pattern can exist for one and the same airfoil (2) with a fixed p.

Unlike the case presented in Fig. 2, the flow structure for other p can change with a hysteresis of the Mach number prescribed at the outlet of the channel. Therefore, different stationary patterns of flow can be realized for one and the same M'_{out} depending on whether a given regime has been obtained with increase in the Mach number or with decrease in it. This result elucidates the physical reasons for the occurrence of nonunique stationary transonic flows near airfoils obtained in the numerical investigations of some authors [2, 10].

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NOTATION

d, distance between supersonic zones; $e(x, y, t)$, internal energy; h_3, height of the third deflection in the shape of the sonic line; M, Mach number; M'_{out}, arbitrary Mach number at the outlet of the channel; M_s, singular Mach number; p and q, parameters in formula (2); p_{out}, pressure at the outlet of the channel; p_0, stagnation pressure; t, time; $u(x, y, t)$ and $v(x, y, t)$, components of the velocity vector in the direction of the x and y axes; x, y, Cartesian coordinates; y_{mid}, thickness of the airfoil at the central point; Δx and Δy, sizes of the grid cells in the central part of the channel; γ, adiabatic exponent; $\rho(x, y, t)$, gas density. Subscripts: max, maximum; mid, central (middle); out, outlet; s, singular; 0, corresponds to the total pressure.

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